THE EXPERIMENTAL METHOD FOR THE STRESS STATE IDENTIFICATION

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1. Introduction

The state of stress in the unloaded surface layer can be only a uniaxial or a plane one. As such, it should be identifiable by measuring strains relieved on the free surface of the prestrained structure during the drilling of a hole perpendicular to the surface. The Hole drilling method for the stress state identification is based on the analytical Kirsch’s stress-state solution [1] of a plate with a hole drilled through perpendicularly and loaded on its x-borders by principal stress \( \sigma_x \). A drilled hole induces a change of the strain state in its close vicinity. These changes of strains arisen by drilling into material can be calibrated with defined principal stresses and subsequently normalised. This relation can be used for an identification of the original stress state in arbitrary loaded structure made from Hook’s material. The Fig. 1 depicts uniaxially loaded specimen with principal stress \( \sigma_x \). The hole of the radius \( R_0 \) is a centre of polar coordinates defined by the ratio radius \( r=R/R_0 \) and angle \( \alpha \). The coordinate \( \Theta \) marks tangential direction at radius \( r \). Symbols \( \sigma'_r, \sigma'_\theta, \tau'_{r\theta} \) stand for stresses in introduced coordinates. The Eq. 1. describes stresses in a specimen without the hole. The Kirsch’s equations (Eq. 2) describe the state of plane stress in the vicinity of the through hole of radius \( R_0 \) (Fig. 1).

\[
\begin{align*}
\sigma'_r &= \frac{\sigma_x}{2}(1 + \cos 2\alpha) \\
\sigma'_\theta &= \frac{\sigma_x}{2}(1 - \cos 2\alpha) \\
\tau'_{r\theta} &= \frac{\sigma_x}{2}\sin 2\alpha 
\end{align*}
\] (1)

\[
\begin{align*}
\sigma'_r &= \frac{\sigma_x}{2}(1 - \frac{1}{r^2}) + \frac{\sigma_x}{2}(1 + \frac{3}{r^2} - \frac{4}{r^2})\cos 2\alpha \\
\sigma'_\theta &= \frac{\sigma_x}{2}(1 + \frac{1}{r^2}) - \frac{\sigma_x}{2}(1 + \frac{3}{r^2})\cos 2\alpha \\
\tau'_{r\theta} &= \frac{\sigma_x}{2}(1 - \frac{3}{r^2} + \frac{2}{r^2})\sin 2\alpha 
\end{align*}
\] (2)

\[
\begin{align*}
\sigma'_r - \sigma'_r &= \frac{\sigma_x}{2}\left(-\frac{1}{r^2}\right) + \frac{\sigma_x}{2}\left(\frac{3}{r^2} - \frac{4}{r^2}\right)\cos 2\alpha \\
\sigma'_\theta - \sigma'_\theta &= \frac{\sigma_x}{2}\left(\frac{1}{r^2}\right) - \frac{\sigma_x}{2}\left(\frac{3}{r^2}\right)\cos 2\alpha \\
\tau'_{r\theta} - \tau'_{r\theta} &= \frac{\sigma_x}{2}\left(-\frac{3}{r^2} + \frac{2}{r^2}\right)\sin 2\alpha 
\end{align*}
\] (3)

\[
\begin{bmatrix}
\varepsilon_r \\
\varepsilon_\theta \\
\gamma_{r\theta}
\end{bmatrix}
= \frac{1}{E}
\begin{bmatrix}
1 & -\nu & 0 \\
-\nu & 1 & 0 \\
0 & 0 & 2(1+\nu)
\end{bmatrix}
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\tau_{r\theta}
\end{bmatrix}
\] (4)

In comparison with Eq. (1), Eq. (2) includes terms dependent on the drilled hole through. Subtraction yields Eq. (3). The right hand side in Eq. 3 can me physically measured through the Eq. 4 using Hook’s law, where \( E \) stands for Young’s modulus and \( \nu \) for Poisson’s ratio. This principle can be used for any isotropic material for a calculation of changes related to strains \( \varepsilon_r, \varepsilon_\theta, \gamma_{r\theta} \) and \( \varepsilon_z \) (see Fig. 1).

2. Regression Models

The measuring properties of the rosettes during drilling the hole according to E 837 standard [2]
are considerably dependent on the accuracy of compliance with standardized conditions of the experiment. If the hole is drilled eccentrically, then the hole drilling experiment, as formulated by E 837 standard, cannot be used for any more complex determination of the strain state in the vicinity of the drilled hole, which would be necessary for any eventual improving corrections. This simple standard drilled theory is not probably reliable for imperfections occurring in drilled holes. In [3], we introduced an experimental evaluation method, which can use data from an arbitrary drilled hole. All stress components are modified by the seven parameters \( c_{ij}(r) \), as described in Eq. (5), analogous to those by Eq. (3), used for a normal straight-through hole. Similar approach is also used by E 837 standard for strain - gage strains, but [3] must not use complete rosettes of strain gauges around the drilling hole. The [3] is capable to use even for a short winding segments. Eq. (5) can be transformed to the strain components using Hooke’s law, similarly as Eq. (3) to Eq. (4).

\[
\begin{align*}
\sigma_\alpha &= \frac{\sigma_x}{r}(-\frac{c_1}{r^2}) + \frac{\sigma_y}{r}(-\frac{3c_2}{r^2} - \frac{4c_3}{r^2}) \cos 2\alpha \\
\sigma_\beta &= \frac{\sigma_x}{2r}(-\frac{3c_1}{r^2}) - \frac{\sigma_y}{2r}(-\frac{3c_3}{r^2}) \cos 2\alpha \\
\tau_\gamma &= \frac{\sigma_x}{2r}(-\frac{3c_2}{r^2} + \frac{2c_4}{r^2}) \sin 2\alpha
\end{align*}
\]

(5)

\[
\begin{align*}
\sigma_{i_1} &= \frac{\sigma_x}{2r}(-\frac{c_{i_1}}{r^2}) + \frac{\sigma_y}{2r}(-\frac{3c_{i_2}}{r^2} + \frac{4c_{i_3}}{r^2}) \sin 2\alpha \\
\sigma_{i_2} &= \frac{\sigma_x}{2r}(-\frac{c_{i_4}}{r^2}) + \frac{\sigma_y}{2r}(-\frac{3c_{i_5}}{r^2}) \sin 2\alpha \\
\tau_{i_2} &= \frac{\sigma_x}{2r}(-\frac{3c_{i_6}}{r^2} + \frac{2c_{i_7}}{r^2}) \sin 2\alpha
\end{align*}
\]

(6)

\[
\begin{align*}
\sigma_{i_3} &= \frac{\sigma_x}{2r}(-\frac{c_{i_1}}{r^2} - \frac{c_{i_2}}{r^2}) + \frac{\sigma_y}{2r}(-\frac{3c_{i_3}}{r^2} - \frac{4c_{i_4}}{r^2}) \cos 2\alpha \\
\sigma_{i_4} &= \frac{\sigma_x}{2r}(-\frac{c_{i_5}}{r^2} + \frac{c_{i_6}}{r^2}) - \frac{\sigma_y}{2r}(-\frac{3c_{i_7}}{r^2}) \cos 2\alpha \\
\tau_{i_4} &= \frac{\sigma_x}{2r}(-\frac{3c_{i_6}}{r^2} + \frac{2c_{i_7}}{r^2}) \sin 2\alpha
\end{align*}
\]

(7)

Analogously, the [4] derives improvement of the measurement sensitivity using the Eq. (6) with seven parameters \( c_{i_1} \ldots c_{i_7} \) for the subsequently drilled holes. The hole-drilling experiment can be repeated with a bigger diameter of the drill, while using the same rosette installed previously, either centrically or eccentrically to the drilled hole. Thus the theory expands the applicability of the hole-drilling principle for the stress state identification. Similarly, in [5] we use the Eq. (7) analogous to Eq. (2) with twelve parameters \( c_{i_1}(r) \ldots c_{i_7}(r) \).

It is proposed for the stress state identification in the surface at the place of already drilled holes with the complete drilling rosette equipment already installed. The method allows a further reusing of already installed measuring items, which were originally placed there for the residual stress state identification, for measurements of the stress states induced by any following external loading as if the hole had not been drilled at all.

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4. References